AFOSR 65-1912-0

AD624080

POLYTECHNIC INSTITUTE OF BROOKLYN

ELECTROPHYSICS DEPARTMENT

Memorandum 96 7 April 1964

PIBMRI-1215-64

MINIMUM-SCATTERING ANTENNAS

by

Walter K. Kahn and Herbert Kurss

ABSTRACT

Antennas with identical patterns differ to the extent in which they modify an incident wave, i.e., in the amount they scatter. An antenna is completely described by an (infinite dimensional) scattering matrix. The concept of a minimum scattering antenna introduced by Dicke is generalized to include antennas with a finite number of accessible waveguide ports and with non-reciprocal components.

A canonical minimum scattering antenna is defined as one which becomes "invisible" when the accessible waveguide terminals are open circuited. Such an antenna is shown to be unique once the independent radiation patterns have been specified. Neither an impedance nor an admittance matrix for such an antenna exists.

The physical significance of the minimum scattering antenna concept is examined from several points of view. Appropriate generalizations of Dicke's results are derived for multiport and non-reciprocal antennas. The "scattered power", is introduced as a convenient measure of scattering. It is demonstrated, for a large class of antennas, that the scattered power is quite generally greater than the absorbed power, equality being attained for minimum scattering antennas of this class. This result further justifies the minimum-scattering terminology. Arrays of canonical antennas are discussed briefly.

The work reported herein was sponsored by the Air Force Office of Scientific Research of the Office of Aerospace Research; the department of the Army, Army Research Office; and the department of the Navy, Office of Naval Research under Grant AF-AFOSR-453-63, and by the National Science Foundation under Grant GP-2761.

MINIMUM-SCATTERING ANTENNAS

I. Introduction

The electromagnetic properties of an antenna are only partially described by its patterns; antennas with identical patterns differ to the extent in which they modify an incident wave, i.e., in the amount they scatter. The concept of a minimum-scattering antenna was first introduced by Dicke[1]. The approach taken in this paper is mathematically much simpler and more general in two respects: It includes antennas with any number of ports and antennas with non-reciprocal components.

The scattering network description of an antenna is introduced in Section II, and a canonical minimum-scattering antenna is defined as one which becomes "invisible" when the accessible waveguide terminals are open circuited. Such an antenna is shown to be unique once the independent radiation patterns have been specified, and that neither an impedance nor admittance matrix for such an antenna exists. The canonical antenna is generalized by cascading it with a "transparent" 2N-port; the result is termed a minimum-scattering antenna.

The physical significance of the minimum-scattering antenna concept is examined from several points of view in Section III. Appropriate generalizations of Dicke's results are derived for multiport and non-reciprocal antennas. The "scattered power", is introduced as a convenient measure of scattering. It is demonstrated, for a large class of antennas, that the scattered power is generally greater than the absorbed power, equality being attained for minimum-scattering antennas of this class. This result further justifies the minimum-scattering terminology. Arrays of canonical antennas are discussed briefly.

II. Canonical Minimum-Scattering Antennas.

The network description of a general antenna is indicated schematically in Fig. 1. The ports at the left, representing the local, accessible waveguide terminals of the antenna, are numbered from 1 to N; the ports on the right, N+1, . . . , representing the electromagnetic fields on a distant sphere enclosing the antenna, are infinite in number. At the N accessible ports normalized, rms-voltage, incident and reflected wave phasors are defined and collected as elements of the column matrices $\underline{a}_a$ and $\underline{b}_a$, respectively, in the conventional manner[1,2]. On a distant sphere the electromagnetic field can be represented as a superposition of a complete, denumerably infinite set of real, orthogonal modes. The modes occur in pairs as incoming and outgoing spherical waves (identified with incident and reflected waves of uniform waveguide terminology). The corresponding modal coefficients, which completely

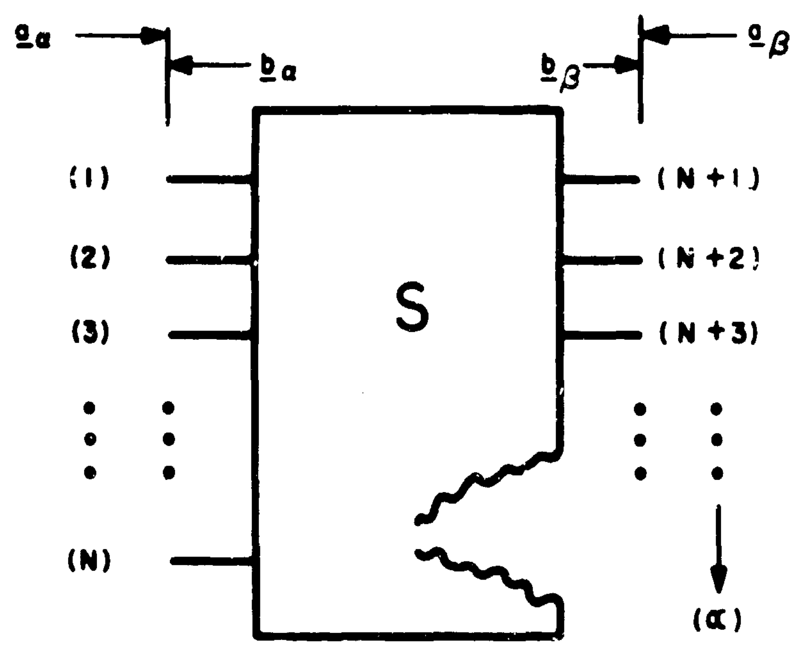

Fig. 1 - A General Antenna

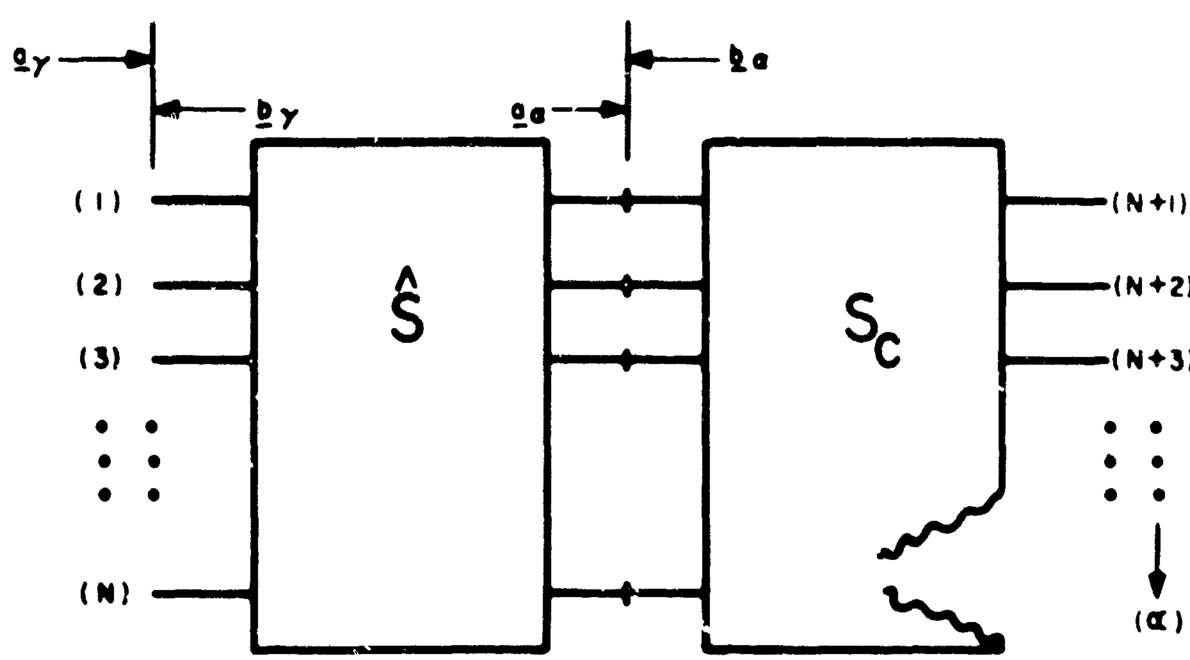

Fig. 2 - Transparent 2N-port in series with a canonical minimum-scattering antenna.

specify the field, are ordered in an arbitrary manner to form the (infinite dimensional) column matrices[1] $\underline{a}_\beta$ and $\underline{b}_\beta$, and normalized* so that $\underline{a}^+_\beta \underline{a}_\beta$ and $\underline{b}^+_\beta \underline{b}_\beta$ are the incident and reflected powers, respectively.

In the absence of any antenna

$$\underline{b}_\beta = \underline{a}_\beta \,. \tag{1}$$

The general (linear) antenna can now be characterized in a scattering formalism

$$\underline{b} = \begin{pmatrix} \underline{b}_\alpha \\ \hline \underline{b}_\beta \end{pmatrix} = \left(\begin{array}{c|c} S_{\alpha\alpha} & S_{\alpha\beta} \\ \hline S_{\beta\alpha} & S_{\beta\beta} \end{array}\right) \begin{pmatrix} \underline{a}_\alpha \\ \hline \underline{a}_\beta \end{pmatrix} \equiv S\,\underline{a} \,. \tag{2}$$

A lossless antenna conserves power, $\underline{a}^+ \underline{a} = \underline{b}^+ \underline{b}$, and consequently, the scattering matrix S is unitary,

$$S^+ S = I \,. \tag{3}$$

As the first step towards an ideal antenna, one demands circuitry such that each accessible port be reflectionless and decoupled from any of the other accessible ports, i.e.,

$$S_{\alpha\alpha} = 0 \,. \tag{4}$$

It follows that the i^{th} column of $S_{\beta\alpha}$ corresponds to the radiation pattern of the antenna when excited at the i^{th} port alone. Condition (4) will hereafter be assumed.

For an evaluation of the (unnecessary) disturbance or scattering of incident fields by an antenna the obvious standard is provided by the absence of any antenna, i.e., free space. In the absence of any antenna the input scattering amplitudes, 1 through N, are meaningless, and corresponding entries in the scattering matrix are therefore omitted. The remainder of the scattering matrix of free space is

$$S_o = \left(\begin{array}{c|c} & \\ \hline & I_{\beta\beta} \end{array}\right) \tag{5}$$

where in virtue of (1) $I_{\beta\beta}$ is an infinite dimensional unit matrix.

A canonical minimum-scattering antenna having specified radiation patterns $S_{\beta\alpha}$ could, therefore, tentatively be defined as a lossless device approaching (1) as closely as possible, suitable adjustments at the accessible ports of the antenna

*Note: $\underline{a}^+$ denotes the conjugate of the transpose of $\underline{a}$.

being permitted. The sense in which (1) is to be approached need not be discussed at this point, since, for any given $S_{\beta\alpha}$ consistent with (3), there exists a unique lossless antenna which on open circuit at ports 1 through N scatters exactly as required. The last statement will now be proved, and the scattering matrix of the canonical antenna found.

On open circuit at ports 1 through N

$$\underline{b}_\alpha = \underline{a}_\alpha \ . \tag{6}$$

Employing (2) to eliminate $\underline{a}_\alpha$ and $\underline{b}_\alpha$ in favor of $\underline{a}_\beta$ and $\underline{b}_\beta$

$$\underline{b}_\beta = (S_{\beta\alpha} S_{\alpha\beta} + S_{\beta\beta}) \ \underline{a}_\beta \ . \tag{7}$$

The scattering relation (7) coincides with the standard, free space (1) and (5), only if

$$S_{\beta\alpha} S_{\alpha\beta} + S_{\beta\beta} = I_{\beta\beta} \ , \tag{8}$$

which may be solved for $S_{\beta\beta}$. Conservation of energy then fixes S uniquely, since (3) is satisfied only if

$$S_{\alpha\beta} = S_{\beta\alpha}^{+} \ . \tag{9}$$

It has therefore been demonstrated that the scattering matrix of the canonical minimum-scattering antenna with radiation patterns $S_{\beta\alpha}$ is

$$S_c = \left(\begin{array}{c|c} 0 & S_{\beta\alpha}^{+} \\ \hline S_{\beta\alpha} & I_{\beta\beta} - S_{\beta\alpha} S_{\beta\alpha}^{+} \end{array}\right) \ . \tag{10}$$

Such an antenna is reciprocal when

$$S_{\beta\alpha}^{T} = S_{\beta\alpha}^{+} \ , \tag{11}$$

i.e., $S_{\beta\alpha}$ is purely real.

Certain purely formal properties of S_c are pointed out next, a full discussion of its physical significance being reserved until Section III.

The eigenvalues of S_c are defined by

$$(S_c - \sigma_n I)\, \underline{s}_n = 0\,, \tag{12}$$

where $\underline{s}_n$ is the eigenvector corresponding to the eigenvalue σ_n. Since S_c is unitary, its eigenvalues lie on the unit circle, $|\sigma_n| = 1$. Since S_c is Hermitean, its eigenvalues are real. Consequently

$$\sigma_n = \pm 1\,, \qquad\qquad n = 1, 2, 3, \ldots\,. \tag{13}$$

The multiplicity of the eigenvalue -1 can be shown to be precisely N. For expanding (12) with $\sigma_n = -1$, one obtains with some manipulation

$$S_{\beta\alpha}^{+}\, \underline{s}_{n\beta} = -\, \underline{s}_{n\alpha}\,; \tag{14a}$$

$$S_{\beta\alpha}\, S_{\beta\alpha}^{+}\, \underline{s}_{n\beta} = \underline{s}_{n\beta}\,. \tag{14b}$$

Equation (14b) implies that $s_{n\beta}$ is in the N-dimensional sub-space spanned by the N columns of $S_{\beta\alpha}$ while (14a) associates a unique $\underline{s}_{n\alpha}$ with each $\underline{s}_{n\beta}$. There are therefore exactly N linearly independent eigenvectors of S_c with eigenvalues -1.

An incidental result which follows directly from (13) is that neither an impedance nor an admittance matrix corresponding to S_c exists.

Consider the effect of inserting a lossless, transparent[3] 2N-port,

$$\hat{S} = \left(\begin{array}{c|c} 0 & \hat{S}_{\gamma\alpha} \\ \hline \hat{S}_{\alpha\gamma} & 0 \end{array}\right) \tag{15}$$

in series with a canonical antenna S_c, as illustrated in Fig. 2. The resultant antenna has the scattering matrix

$$\hat{S}_c = \left(\begin{array}{c|c} 0 & \hat{S}_{\gamma\alpha} S_{\beta\alpha}^{+} \\ \hline S_{\beta\alpha} \hat{S}_{\alpha\gamma} & I_{\beta\beta} - S_{\beta\alpha} S_{\beta\alpha}^{+} \end{array}\right). \tag{16}$$

Antennas characterized by scattering matrices of the form (16) will be termed simply minimum-scattering antennas. This terminology is justified by the fact that such an antenna is rendered invisible by a particular lossless termination of the accessible

waveguide ports. Further, when reflectionless loads (receivers) are attached at ports 1 through N, S_c and $\hat{S}_c$ scatter identically ($\hat{S}_{\beta\beta} = S_{\beta\beta}$). In the special case

$$S_{\alpha\gamma} = S^{+}_{\gamma\alpha} \tag{17}$$

$\hat{S}_c$ is also a canonical minimum-scattering antenna.

III. Characteristics of Minimum-Scattering Antennas

The scattering matrix of a canonical minimum-scattering antenna having N specified (orthogonal) radiation patterns was constructed in the preceding section. The construction was found to be unique. In this section the physical significance of this construct is examined from several points of view.

When an electromagnetic wave $\underline{a}$

$$\underline{a} = \begin{pmatrix} 0 \\ \underline{a}_\beta \end{pmatrix} \tag{18}$$

is incident on an antenna S, waves $\underline{b}_\alpha = S_{\alpha\beta}\,\underline{a}_\beta$ are excited in the N waveguide ports of the antenna and waves $\underline{b}_\beta = S_{\beta\beta}\,\underline{a}_\beta$ are reflected into the surrounding space. The scattered field $\underline{f}_\beta$ due to an antenna is the difference in the external fields when an antenna is present as compared to the situation when no antenna is present. Formally,

$$\underline{f}_\beta = \underline{b}_\beta - \underline{a}_\beta \ , \tag{19a}$$

or in the special case of a matched antenna, $\underline{a}_\alpha = 0$,

$$\underline{f}_\beta = (S_{\beta\beta} - I_{\beta\beta})\,\underline{a}_\beta \ . \tag{19b}$$

As a definite measure of these scattered fields one may introduce the concept of scattered power

$$P_S = |\underline{f}_\beta|^2 \ . \tag{20}$$

For a canonical minimum-scattering antenna (10) with matched receivers, one verifies

$$P_S = |\,(S_{\beta\beta} - I_{\beta\beta})\,\underline{a}_\beta\,|^2 = P_A \tag{21}$$

where $P_A = |b_\alpha|^2$ is the absorbed power. Moreover, the pattern of this scattered radiation is a linear combination of the orthogonal patterns in which the antenna normally radiates, $\underline{f}_\beta = - S_{\alpha\beta}\, \underline{b}_\alpha$. These results carry over to minimum-scattering antennas in general.

The physical mechanism through which an antenna absorbs power from an incident wave is destructive interference, i.e., the antenna scatters so as to cancel some of the incident fields. It may be conjectured, therefore, that, quite generally,

$$P_S \geq P_A \,. \tag{22}$$

This is the case for all antennas with

$$|S_{\alpha\beta}\, \underline{a}_\beta| = |S^{+}_{\beta\alpha}\, \underline{a}_\beta| \tag{23}$$

as demonstrated in the Appendix. Equation (23) holds for all real excitations of arbitrary reciprocal antennas or arbitrary excitation of reciprocal antennas with real patterns*. For minimum-scattering antennas equality is achieved in equation (22).

When the canonical minimum-scattering antenna is reciprocal, S_c is pure real,(11), and several significant new features appear. The first has to do with the symmetry of the antenna patterns. Given an incident plane wave, the function representing a second wave incident from the opposite direction is the complex conjugate of the first. Since the spherical mode functions are real, the modal coefficients representing the second wave $\underline{a}_{\beta 2}$ are the complex conjugates of the coefficients representing the first $\underline{a}_{\beta 1}$, i.e.,

$$\underline{a}_{\beta 2} = \underline{a}^{*}_{\beta 1} \,. \tag{24}$$

The power absorbed in the two cases are equal

$$|b_{\alpha 1}|^2 = |S_{\alpha\beta}\, \underline{a}_{\beta 1}|^2 = |S_{\alpha\beta}\, \underline{a}^{*}_{\beta_1}|^2 = |\underline{b}_{\alpha 2}|^2 \,. \tag{25}$$

It follows that the power radiation patterns of such an antenna are symmetrical with respect to the origin. An elementary dipole illustrates these features.

Consider the possibility that the scattering matrix of an antenna varies with frequency in such a manner as to retain the form of $S_c = S^{*}_c$. From the general

---

*c.f. equation (25).

theorem[4] relating the frequency variation of a lossless reciprocal junction to the average energy stored in the junction W

$$\underline{a}^{+}\left[j\, S_c^{*}\ \frac{dS_c}{d\omega}\right]\underline{a} \geq W \tag{26}$$

for arbitrary $\underline{a}$, it follows that W = 0, and hence that

$$\frac{dS_c}{d\omega} = 0\,. \tag{27}$$

Finally a brief remark may be entered regarding arrays of canonical minimum-scattering antennas. Assume the elements to be so spaced that the spheres circumscribed about each element do not overlap. Each element, on open circuit, produces no disturbance in any incident field, and therefore the array as a whole is invisible. Nevertheless, the array is not necessarily a canonical minimum-scattering antenna, since the orthogonality relations (3) are not necessarily satisfied Prescribed feed networks may to some degree of approximation assure this orthogonality. The properties of an array of such antennas are peculiarly susceptible to exact computation:

a) The fields radiated by a given element (with all others open circuited) are rigorously the same as those which would be radiated by the element isolated and

b) The open circuit voltage computed from this field at each one of the other elements is rigorously that which would be computed for an isolated antenna.

In conclusion, the analysis of minimum-scattering antennas has been greatly simplified and the range of this concept considerably extended beyond results previously published.

Appendix: A Relation Between P_A and P_S.

Let

$$\underline{a} = \begin{bmatrix} 0 \\ \\ \underline{a}_\beta \end{bmatrix} \quad , \quad \underline{b} = \begin{bmatrix} \underline{b}_\alpha \\ \\ \underline{b}_\beta \end{bmatrix}$$

and define the scattered power, P_S, and the absorbed power, P_A,

$$P_S = |\underline{b}_\beta - \underline{a}_\beta|^2 \;;\; P_A = |\underline{b}_\alpha|^2 \;.$$

For a lossless antenna conservation of energy requires

$$|\underline{a}_\beta|^2 = |\underline{b}_\alpha|^2 + |\underline{b}_\beta|^2 \,.$$

Hence,

$$\begin{aligned} P_S &= |\,\underline{b}_\beta\,|^2 + |\underline{a}_\beta|^2 - 2\,\mathrm{Re}\,(\underline{a}_\beta^+\ \underline{b}_\beta) \\ &= P_A + 2\left[|\underline{b}_\beta|^2 - \mathrm{Re}\,(\underline{a}_\beta^+\ \underline{b}_\beta)\right] \;. \end{aligned}$$

Thus $P_S \geq P_A$ if and only if

$$\mathrm{Re}\,(\underline{a}_\beta^+\ \underline{b}_\beta) \leq |\underline{b}_\beta|^2 \;.$$

Clearly it is sufficient to show that

$$|(\underline{a}_\beta^+\ \underline{b}_\beta)| \leq |\underline{b}_\beta|^2 \;.$$

Consider the determinant of the Gram[5] matrix formed from $\underline{a}_\beta$, $\underline{b}_\beta$, and the N columns of $S_{\beta\alpha}$, i.e.,

$$\det \begin{bmatrix} \underline{a}_\beta^+ \underline{a}_\beta & \underline{a}_\beta^+ \underline{b}_\beta & \underline{a}_\beta^+ S_{\beta\alpha} \\ \\ \underline{b}_\beta^+ \underline{a}_\beta & \underline{b}_\beta^+ \underline{b}_\beta & \underline{b}_\beta^+ S_{\beta\alpha} \\ \\ S_{\beta\alpha}^+ \underline{a}_\beta & S_{\beta\alpha}^+ \underline{b}_\beta & S_{\beta\alpha}^+ S_{\beta\alpha} \end{bmatrix} \geq 0 \,.$$

From the lossless constraint

$$S^+_{\beta\alpha}\, \underline{b}_\beta = 0 = \underline{b}^+_\beta\, S_{\beta\alpha} ,$$

and

$$S^+_{\beta\alpha}\, S_{\beta\alpha} = I_N .$$

Hence the above determinantal inequality leads to

$$|(\underline{a}^+_\beta \quad \underline{b}_\beta)|^2 \leq |\underline{b}_\beta|^2 \; (|\underline{a}_\beta|^2 - |S^+_{\beta\alpha}\, \underline{a}_\beta|^2) \; .$$

If now, equation (23),

$$|S_{\alpha\beta}\, \underline{a}_\beta| = |S^+_{\beta\alpha}\, \underline{a}_\beta| ,$$

is satisfied,

$$|(\underline{a}^+_\beta \quad \underline{b}_\beta)|^2 \leq |\underline{b}_\beta|^2 \,(|\underline{a}_\beta|^2 - |\underline{b}_\alpha|^2) ,$$

or, in view of conservation of energy,

$$(\underline{a}^+_\beta \; \underline{a}_\beta) \leq |\underline{b}_\beta| \; .$$

Thus (23) is a sufficient condition for the validity of $P_S \geq P_A$ for arbitrary $\underline{a}_\beta$.

References

1. C.G. Montgomery, R.H. Dicke, and E.M. Purcell, "Principles of Microwave Circuits," Radiation Laboratory Series Vol. 8, McGraw-Hill Book Co., Inc., 1948, pp. 317-333.

2. N. Marcuvitz, "Waveguide Handbook," Radiation Laboratory Series Vol. 10, McGraw-Hill Book Co., Inc., 1951, Section 2.8.

3. W.K. Kahn, "Scattering Equivalent Circuits for Common Symmetrical Junctions," I.R.E. Transactions on Circuit Theory, Vol. CT-3, No. 2, June 1956, pp. 121-127.

4. Ibid. 1, p. 154.

5. F.R. Gantmacher, "The Theory of Matrices," Chelsea Publishing Company, New York, 1959, Vol. II, p. 251.